MOTOR TIME CONSTANT... Pastel

Marvin Paul Pastel

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UNITED STATES
NAVAL POSTGRADUATE SCHOOL

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While various methods of derivative feedback are employed to reduce the effective time constant of motor type instrument integrators, costly overdesign or significant error may result if an error analysis is not made.

The transfer function diagram of an instrument integrator is shown in Figure (1a) where the value of K corresponding to the scale factor is included for completeness and p is the Heaviside operator ($p = d/dt$). The output quantity represents a perfect integration of the input quantity as modified by the scale factor. Unfortunately, practical integrators are plagued by various sources of errors such as motor friction, non-linear characteristics, noise and the motor time constant.

The latter source of error, which will be of primary concern here, differs from the other three by changing the linear transfer function representation of the instrument to that shown in Figure (1b).

Time Constant Error

The presence of this time constant results in an integration error with time which may be computed by taking the difference between the true output and the actual output, i.e.:

$$e(t) = \theta(t) - \phi(t) \tag{1}$$

where

$$\theta(t) = \frac{K}{p} i(t)$$
$$\phi(t) = \frac{K}{p(1+Tp)} i(t) \tag{2}$$

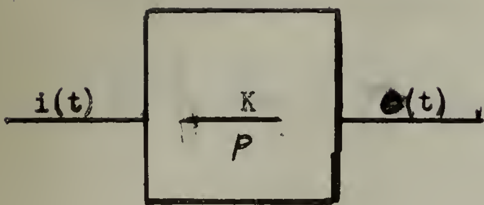

Figure 1a

Linear ideal integrator

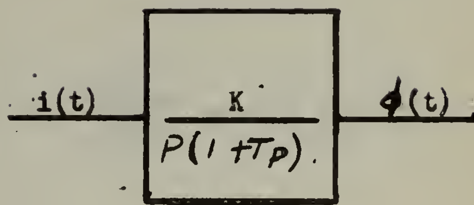

Figure 1b

Linear integrator with time constant

To develop a general formula for the error function permitting analysis of the error for an arbitrary input, use of the Laplace transform techniques will be necessary. However, several simple input time functions will be illustrated.

Rearranging and taking the Laplace transform of (2) yields:

$$s\Theta(s) - \theta(0) = KI(s)$$

$$s\Phi(s) - \phi(0) + Ts^2\Phi(s) - Ts\phi(0) - T\frac{d\phi(0)}{dt} = KI(s) \quad (3)$$

Since the initial output can be made zero by translation of the axis, the following initial conditions should be general enough to cover most applications.

$$t = 0$$

$$\theta(0) = 0$$

$$\phi(0) = 0$$

$$\frac{d\phi(0)}{dt} = v \quad (4)$$

Substituting these values into (3) and solving for the inverse transform of $\Theta(s)$ and $\Phi(s)$ gives:

$$\theta(t) = L^{-1}\left[\frac{K}{s} I(s)\right]$$

$$\phi(t) = L^{-1}\left[\frac{KI(s)}{s(1+Ts)} + \frac{Tv}{s(1+Ts)}\right] \quad (5)$$

$$= L^{-1}\left[\frac{KI(s)}{s} - \frac{KI(s)}{1/T + s}\right] + Tv\left(1 - e^{-t/T}\right) \quad (6)$$

substituting (5) and (6) into (1) yields:

$$e(t) = L^{-1}\left[\frac{K\,I(s)}{1/T + s}\right] - Tv\left(1 - e^{-t/T}\right) \tag{7}$$

For a known input, the Laplace transform may be taken, substituted into (7) and the resulting inverse transform will yield the error caused by the time constant. This has been done for the step displacement and ramp (step velocity) time input functions. The results are given in the accompanying table of errors and velocity functions.

The functions for output velocity are directly obtainable by differentiating of (6). These are useful in establishing initial condition when a complex input function is approximated by step and ramp functions. This method will be illustrated later. When time is large the simpler error and velocity equations may be used with less than 1% error.

An Example:

The formulas in Table I can be used to find the approximate error in a missile guidance system caused by the time constant in the velocity integrator. Figure (2) represents a possible variation of missile velocity which has been approximated by straight lines.

K will be considered as unity and the physical variable used instead of the analog variable, i.e., current or voltage. If T has the value of 10^{-2} the equation for large time may be used giving an error of 100 ft and output velocity of 10,000 ft/sec at 80 seconds where the assumed

Table 1 - Error and Velocity Functions

	Step Input $i(t) = a$	Ramp Input $i(t) = at$
Error $e(t)$	$(aKT - Tv)(1 - e^{-t/T})$ (8)	$(T^2 aK + Tv)(e^{-t/T} - 1) + T^2 aK(t/T)$ (12)
Error $e(t)$ Time Large $t/T > 5$	$aKT - Tv$ (9)	$TaKt - Tv - aKT^2$ (13)
Output Velocity $v(t)$	$Ka + (v - Ka)e^{-t/T}$ (10)	$Kat - TKa + (TKa + v)e^{-t/T}$ (14)
Output Velocity $v(t)$ Time Large $t/T > 5$	Ka (11)	$Ka(t - T)$ (15)

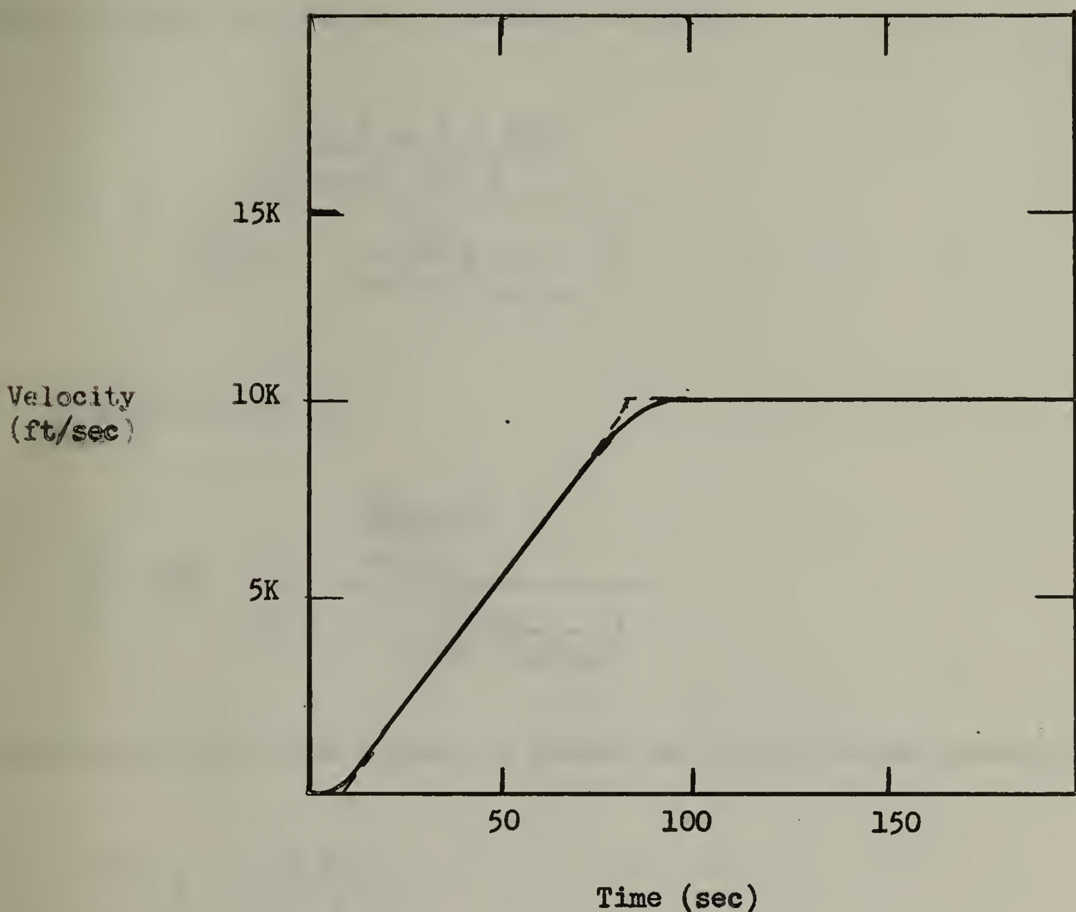

Figure 2: Missile Velocity Variation With Time

velocity curve levels off. Since the output velocity of the integrator quantity corresponds to the input quantity velocity, no further error is introduced.

Motor Tachometer Integrator

The Tachometer feedback servo type integrator is used so often as to warrant a more complete analysis. The transfer function diagram representation of Figure (3) has for a transfer function:

$$\frac{\phi}{i} = \frac{\dfrac{K_a K_m K_n K_p}{K_1 K_t K_a K_m + 1}}{s\left(1 + \dfrac{T_m s}{1 + K_1 K_t K_a K_m}\right)} \tag{16}$$

If $K_1 K_t K_a K_m \gg 1$

$$\frac{\phi}{i} \cong \frac{\dfrac{K_n K_p}{K_1 K_t}}{s\left(1 + \dfrac{T_m s}{K_1 K_t K_a K_m}\right)} \tag{17}$$

Comparison of (17) with Figure (1) points out the following correspondance

$$K = \frac{K_n K_p}{K_1 K_t} \tag{18}$$

$$T = \frac{T_m}{K_1 K_t K_a K_m} \tag{19}$$

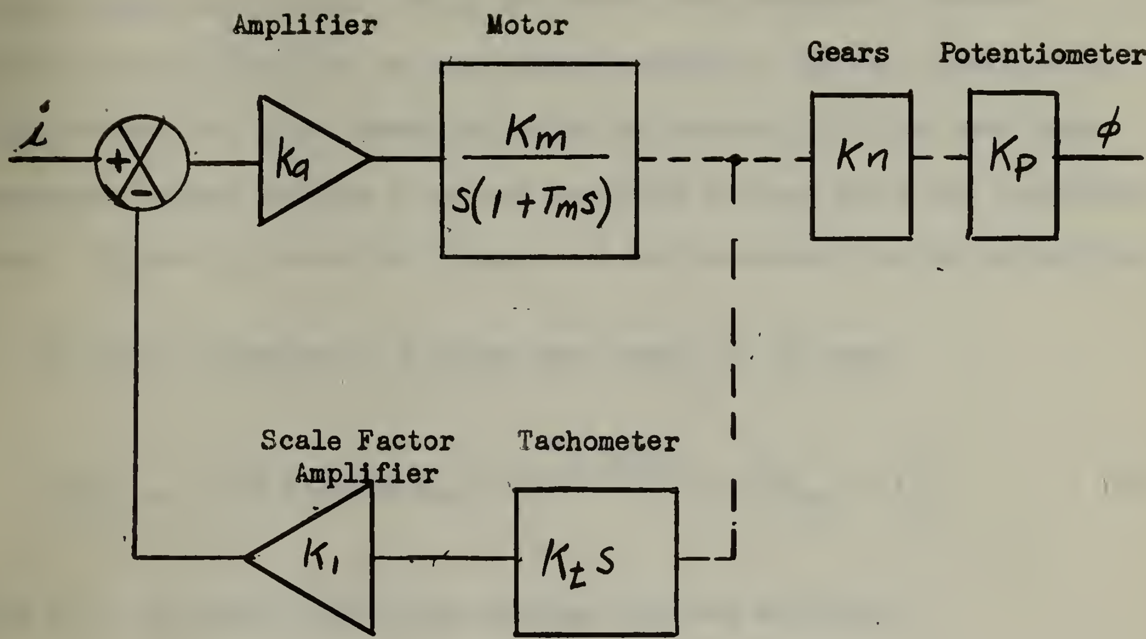

Figure 3

Motor Tachometer Type Integrator

These relationships show that tachometer feedback improves the motor integrator performance by reducing the time constant. The scale factor is a function of the reciprocal of the feedback constants.

Since high gain amplifiers are normally employed, a step input will usually cause saturation. While saturated, the tachometer feedback voltage has no effect on the integrator operation. Linear operation is restored when the motor speed builds up sufficiently for the resulting tachometer output voltage to reduce the error voltage below the saturation level. Figure (4) shows the dynamics of the instrument during saturation.

The output function to a large step input, a, is then:

$$\phi(t) = T_m K_n K_m K_p D\left(e^{-t/T_m} + t/T_m - 1\right) \tag{20}$$

When D is the saturation output voltage from the amplifier.

During saturation, the error is given by

$$e(t) = aKt - T_m K_n K_m K_p D\left(e^{-t/T_m} + t/T_m - 1\right) \tag{21}$$

The instrument leaves the saturated state and becomes linear when

$$a - \frac{K_1 K_T v}{K_n} = \frac{D}{K_a} \tag{22}$$

The output rate of change of the integrator while saturated is:

$$\frac{d\phi(t)}{dt} = K_n K_m K_p D\left(1 - e^{-t/T_m}\right) \tag{23}$$

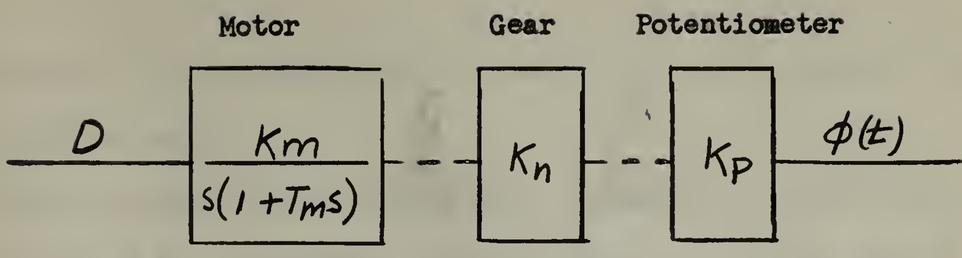

Figure 4

System of Figure 3 During Saturation

Substituting the value of v from (22) into (23) and solving for t yields the time when the instrument leaves saturation.

$$t = -T_m \operatorname{Log}\left[1 - \frac{1}{K_m K_p D K_1 K_T}\left(a - \frac{D}{K_a}\right)\right] \tag{24}$$

This value of t when substituted in (21) will give the error at end of the saturation state.

The value of output rate of change from (22) or (23) may be used as the value of v and input saturation limit, D/K_a may be used for the value of a for (8) or (9) in Table I for determining the error when the instrument is linear. The sum of these two errors represents the total error from the large step input.

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